brought in from an RMU or may be found within the computer. In the latter event, PAC accesses the processor memory and pulls out the desired data for transmission. The format is flexible and permits desired sampling frequencies to be attained.

The CDP performs the tasks of navigation, guidance, flight control, attitude control, self check, and system malfunction detection. It operates as a sampled data system using inputs brought in as a result of its own periodically made request, and also those periodic inputs automatically brought in by the PAC. After processing, outputs are sent via the PAC to the ROUs to control the engine action.

Advantages

The system offers three distinct advantages; growth capability, noise immunity, and high efficiency in terms of size, weight, and power.

Growth or modification capability is provided by the modular design of the system. The central exchange electronics package (PAC) is the primary component while the central processor and the remainder of the system elements are merely plug-in units. The interface between the PAC and the central processor is quite standard so that central processors may be changed without a major PAC redesign.

The PAC unit acquires data by addressing the remote multiplexer units and sends digital data to the remote output units. The digital-to analog conversion is made at the device to be driven. Figure 2, a sketch of the system operating in a typical booster, illustrates the location of the RMU and ROU units. They are placed relatively near the source or destination of the information. The resulting digital transmission of the input and output data has several advantages. The radiated noise environment through which the cables run has little or no effect upon a digital signal. Hence, cleaner signals are obtained from the sensors used to gather data. Another important advantage is provided by party-line communication, i.e., each RMU can be used to gather ininformation from several sources as long as they are physically close together. The sensor outputs are converted to digital at the source and sequentially sent down the same line to the PAC unit. Therefore, each sensor does not require its own communication line. This same party line philosophy is used by the ROUs.

System Simulation

The flight system has been breadboarded and is in operation at the Martin Marietta Denver Inertial Laboratory as illustrated by Fig. 3. An IBM 4π CP2 computer acts as the CDP.

A six-degree-of-freedom, real-time trajectory and an IMU model are simulated in a laboratory computer (DEC PDP-9) and used to provide closed loop verification of the airborne electronics guidance and control system. The simulation is designed to exercise all flight functions of the airborne system.

As shown in Fig. 3, the engine actuator commands, vehicle sequencing discretes, and the commands to the attitude control system engines are input to the PDP-9 simulation. The A/D and D/A converters shown are within the PDP-9 simulation. The A/D and D/A converters shown are within the PDP-9. The PDP-9 calculates the vehicle motion due to the forces acting on it and generates analog signals to simulate roll, pitch, and yaw rate gyros, inertial measurement unit gimbal angles and accelerometer outputs. In addition, the PDP-9 calculates the vehicle state variables in an inertial reference frame.

Since the accelerometer and gimbal angle information is in digital form, it goes directly into the PAC unit that has been constructed as a breadboard model. The PAC stores this information in the 4π CP2 memory on a cycle-stealing basis. The PAC unit also commands an RMU to convert the analog sensor information (i.e., rate gyro signals) to digital words and causes this information to be stored in the 4π CP2 memory.

The 4π CP2 has been programed to perform the guidance and flight control functions and thereby provide control of the vehicle in all phases of flight. This is accomplished by vectoring the engine thrust during powered flight and on-off control of the attitude control engines during coast flight in response to attitude error steering commands.

The required engine actuator displacement or attitude control system status is achieved via the PAC and an ROU. For example, during powered flight, engine actuator displacements are calculated by the 4π CP2. The 4π CP2 then sends an address and the displacement information to the PAC. The PAC decodes the address and causes the proper channel of the proper ROU to convert the digital displacement command to an analog signal to drive the actuator.

The airborne electronics guidance and control system is programed to pilot the booster on a satellite injection mission. The satellite injection mission places multiple payloads into synchronous equatorial orbit. The entire boost mission requires $\sim$6.5 hr. The demonstration of the satellite injection mission consists of simulation of four stages of powered flight, the associated staging sequences, and all phases of attitude control system operation.

Conclusions

The system described exploits the advantages of an all-digital system and permits the tailoring of the electronics to the unique and multifarious problems of a large flexible booster. The system performs the logical computation, data acquisition, and data distribution efficiently and in an optimum mode. Requirement changes can be accomplished in a straightforward, logical, inexpensive manner. This is accomplished by use of the building block concept.

A single hardware system performs a variety of functions through efficient time-sharing of hardware and eliminates equipment duplication. Small changes in requirements can be accommodated by redesign of a small part of the system and not by redesign of major system elements. The concept allows major reconfiguration with little or no redesign or requalification of system elements. This is accomplished without the significant penalty of excess hardware for even the smallest set of requirements.

Galerkin Stress Functions

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STRESS analysis techniques that utilize closed-form solutions of the governing equations, but only approximately satisfy the boundary conditions, are coming into their own in engineering applications as one alternative to the more established finite element and finite-difference techniques. For example, a least-squares point matching method has been applied recently[1] to axisymmetric problems of incompressible solid-propellant rocket charges; a system of two biharmonic Galerkin stress functions was suggested for the extension of the method to star-centered charges, which possess n-fold

Received September 11, 1969; revision received December 4, 1969. Summerfield Research Station is operated on behalf of the Ministry of Technology by Imperial Metal Industries Limited, a subsidiary company of Imperial Chemical Industries Ltd.

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axial symmetry. It is feasible that trial solutions of this problem could consist of the sum of axisymmetric solutions and solutions that are periodic in the azimuthal angle; biharmonic functions with these properties are well known.

A formulation that utilizes as many standard solutions as possible is essential to advanced applications of techniques that rely on closed-form solutions. The Galerkin stress functions probably admit more standard solutions than any other formulation, but the literature apparently lacks a simple direct derivation of these functions under conditions that are just sufficiently general to cover engineering applications. Ordinarily, stress functions are obtained inversely, but recently De La Penha and Childs[2] derived the axisymmetric Love stress function directly from the equations of equilibrium. In this Note the Galerkin stress functions will be derived directly from Navier's equations for the fully three-dimensional case, assuming a finite body. The Love stress function emerges as a special case for axial symmetry in cylindrical coordinates.

Let the body occupy a finite region, R, bounded by an outer closed surface which may contain inner closed surfaces. Let the displacement vector, $\mathbf{u}$, be a point function that, along with its derivatives, is single valued, finite and continuous in R and vanishes outside R. With $\alpha = 2(1-\nu)/(1-2\nu)$ (ν = Poisson's ratio), the homogeneous parts of Navier's equations[3] may be written

$$\nabla^2\mathbf{u} + (\alpha - 1)\text{ grad div}\mathbf{u} = \mathbf{0} \tag{1}$$

By Helmholtz's theorem[4] there exist ϕ and $\mathbf{V}$ such that

$$\mathbf{u} = A(\text{grad}\phi - \alpha\text{ curl}\mathbf{V}) \tag{2}$$

where A ($\neq 0$) is an arbitrary constant. The representation Eq. (2) is not unique, because ϕ is defined only up to a constant and $\mathbf{V}$ is defined only up to a gradient vector. So without loss of generality, put

$$\mathbf{V} = \text{curl}\mathbf{U} \tag{3}$$

It is required to express ϕ generally in terms of $\mathbf{u}$. Evidently $\mathbf{u}$ is unaffected by the addition of an arbitrary gradient vector to $\mathbf{U}$. This gradient vector can be chosen so that

$$\phi = \text{div}\mathbf{U} \tag{4}$$

for if $\mathbf{U} = \mathbf{U}' + \text{grad}W$, then $\phi = \text{div}\mathbf{U}' + \nabla^2 W$ and the validity of Eq. (4) rests on the solution of Poisson's equation. From the conditions imposed on $\mathbf{u}$ and the relationships of ϕ and $\mathbf{U}$ to $\mathbf{u}$, ϕ and div$\mathbf{U}'$ must be finite and continuous in R and may be set equal to zero outside R, hence the standard solution,

$$W = (4\pi)^{-1} \iiint_R r^{-1}(\text{div}\mathbf{U}' - \phi)dV$$

is valid. Equations (2–4) give

$$\mathbf{u} = A(\text{grad div}\mathbf{U} - \alpha\text{ curl curl}\mathbf{U})$$

hence

$$\mathbf{u} = A[\nabla^2\mathbf{U} - (\alpha - 1)\text{ curl curl}\mathbf{U}] \tag{5}$$

On substituting $\mathbf{u}$ from Eq. (5), Eq. (1) reduces to $A\,\alpha\,\nabla^4\mathbf{U} = \mathbf{0}$. The case $\nu = \frac{1}{2}$ may be included by choosing $A = 2\nu - 1$, so for $\nu \neq 1$

$$\nabla^4\mathbf{U} = \mathbf{0} \tag{6}$$

and Eq. (5) becomes

$$\mathbf{u} = \text{grad div}\mathbf{U} - 2(1-\nu)\nabla^2\mathbf{U} \tag{7}$$

Equation (7) [subject to Eq. (6)] is the Galerkin solution. In Cartesian coordinates the components of Eq. (6) are the biharmonic Galerkin stress functions. In cylindrical coordinates, r, θ, z with unit vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, respectively, let $\mathbf{U} = U_1\,\mathbf{a} + U_2\,\mathbf{b} + U_3\,\mathbf{c}$. Then

$$\text{div}\mathbf{U} = \partial U_1/\partial r + U_1/r + r^{-1}\partial U_2/\partial\theta + \partial U_3/\partial z$$

and

$$\nabla^2\mathbf{U} = \{\nabla^2 U_1 - U_1/r^2 - 2r^{-2}\partial U_2/\partial\theta\}\mathbf{a} + \{2r^{-2}\partial U_1/\partial\theta + \nabla^2 U_2 - U_2/r^2\}\mathbf{b} + \nabla^2 U_3\mathbf{c}$$

So if $U_1 = U_2 = \partial U_3/\partial\theta = 0$, the solution reduces to the axisymmetric solution with U_3 as the Love stress function.

The conditions imposed on R and $\mathbf{u}$ are not the most general for which the Galerkin solution is valid, but the preceding direct derivation applies to most situations of practical interest. The transformation[3] of the Galerkin solution to the Neuber–Papkovich solution is insufficient to establish generality, because the expression

$$\mathbf{u} = \text{grad div}\mathbf{U} - (1 - 2\nu)\text{ grad}F - 2(1-\nu)\nabla^2\mathbf{U}$$

also satisfies Eq. (1) provided that $\nabla^2(\nabla^2\mathbf{U} + \text{grad}F) = \mathbf{0}$, and similarly may be transformed to the Neuber–Papkovich solution. It is not obvious that F is disposable. For $\nu = \frac{1}{2}$, $\mathbf{u}$ does not contain F, but div$\mathbf{u} = (2\nu - 1)\nabla^2(\text{div}\mathbf{U} + F)$, and the limit as $\nu \to \frac{1}{2}$ of div$\mathbf{u}/(1-2\nu)$ occurs in the linear expressions for the direct stresses.

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One-Dimensional Theory of Monopropellant Rocket Combustion

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Nomenclature

A = $\dot{m}''_f rc/k \ln(1+B)$
B = transfer number, $c(T_b - T_s)/Q$
a,b = coefficients in ξ^* expression, Eq. (11)
c = constant pressure heat capacity
D = diameter of the droplet
G = mass flow rate at injection per unit duct area
k = thermal conductivity of the gas
$\dot{m}''_f$ = mass flow rate per unit area in a plane laminar flame
$\dot{m}''_s$ = mass flow rate per unit area at the droplet surface
Pr = Prandtl number
P = exponent in ξ^* expression, Eq. (11)
Q = latent heat of vaporization of the propellant
Re_0 = Reynolds number, $2Gr_0/\mu$
r = droplet radius
T_b = adiabatic burnt temperature
T_s = droplet surface temperature
t = time
u,v = gas velocity and droplet velocity, respectively
W = $\rho u/G$
x = axial distance
ζ = r/r_0
μ = viscosity of the gas

Received December 10, 1969. The authors thank H. S. Mukunda for discussions and the authorities of TIFR, Bombay, for the CDC-3600 Computer facility provided.

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